

Problem Set 6.2

#2-38 even

$$2) \frac{\sqrt{2} \csc \theta}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$\csc \theta = \frac{2}{\sqrt{2}}$$

$$\sin \theta = \frac{\sqrt{2}}{2} \quad \begin{array}{|c} \sqrt{45^\circ} \\ \hline \end{array}$$



$$\theta = 45^\circ, 135^\circ$$

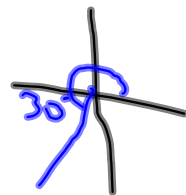
$$4) \quad 2\sqrt{3} \sec \theta + 7 = 3$$

$$\frac{2\sqrt{3} \sec \theta}{2\sqrt{3}} = \frac{-4}{2\sqrt{3}}$$

$$\sec \theta = -\frac{2}{\sqrt{3}}$$

$$\sec \theta = -\frac{2}{\sqrt{3}}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$



$$\theta = 150^\circ, 210^\circ$$

$$6) 4\cos\theta - 3\sec\theta = 0$$

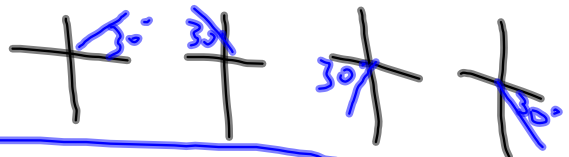
$$\cos\theta \left[4\cos\theta - \frac{3}{\cos\theta} = 0 \right]$$

$$4\cos^2\theta - 3 = 0$$

$$\theta \neq 90^\circ, 270^\circ$$

$$\frac{4\cos^2\theta = 3}{4 \quad 4}$$
$$\sqrt{\cos^2\theta} = \sqrt{\frac{3}{4}}$$

$$\cos\theta = \pm \frac{\sqrt{3}}{2}$$



$$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

$$8) \csc \theta + 2 \cot \theta = 0$$

$$\sin \theta \left[\frac{1}{\sin \theta} + \frac{2 \cos \theta}{\sin \theta} = 0 \right]$$

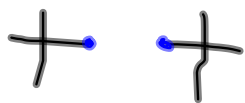
$$\frac{1 + 2 \cos \theta = 0}{-1 \quad -1} \quad \theta \neq 0^\circ, 180^\circ$$


$$\frac{2 \cos \theta = -1}{2} \quad \frac{60^\circ}{\cancel{\theta}}$$

$$\cos \theta = -\frac{1}{2} \quad \frac{60^\circ}{\cancel{\theta}}$$

$$\theta = 120^\circ, 240^\circ$$

$$10) \quad 2 \sin \theta + \sin 2\theta = 0$$
$$2 \sin \theta + 2 \sin \theta \cos \theta = 0$$
$$2 \sin \theta (1 + \cos \theta) = 0$$

$$\sin \theta = 0$$
A horizontal number line with two tick marks. Each tick mark has a small blue dot above it, representing the solutions to the equation sin theta = 0.

$$\cos \theta = -1$$
A horizontal number line with two tick marks. Each tick mark has a small blue dot above it, representing the solutions to the equation cos theta = -1.

$$\theta = 0^\circ, 180^\circ$$

$$12) 2 \sin \theta - 1 = \csc \theta$$

$$\sin \theta \left[2 \sin \theta - 1 = \frac{1}{\sin \theta} \right]$$

$$\frac{2 \sin^2 \theta - \sin \theta = 1}{-1 \quad -1}$$

$$\theta \neq 0, 180^\circ$$

$$2 \sin^2 \theta - \sin \theta - 1 = 0$$

$$x = \sin \theta$$

$$2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0$$

$$x = -\frac{1}{2}, 1$$

$$\sin \theta = -\frac{1}{2}$$

$$\theta = 210^\circ, 330^\circ$$

$$\sin \theta = 1$$

$$\theta = 90^\circ$$

$$\theta = 210^\circ, 330^\circ, 90^\circ$$

$$14) \cos 2x - \cos x - 2 = 0$$

$$2\cos^2 x - 1 - \cos x - 2 = 0$$

$$2\cos^2 x - \cos x - 3 = 0$$

$$x = \cos x$$

$$2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$x = \frac{3}{2}, -1$$

~~$$\cos x = \frac{3}{2}$$~~

$$\cos x = -1$$

$$x = 180^\circ$$

$$x = \pi$$

$$16) \sin x = -\cos 2x$$

$$\sin x = -(1 - 2\sin^2 x)$$

$$\sin x = -1 + 2\sin^2 x$$

$$\begin{array}{r} -\sin x \\ \hline -\sin x \end{array}$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$x = \sin x$$

$$2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0$$

$$x = -\frac{1}{2}, 1$$

$$\sin x = -\frac{1}{2}$$

$$\cancel{30^\circ} \quad \cancel{150^\circ}$$

$$\sin x = 1$$

$$x = 210^\circ, 330^\circ, 90^\circ$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$$

$$18) 2 \sin^2 x - \cos x - 1 = 0$$

$$2(1 - \cos^2 x) - \cos x - 1 = 0$$

$$2 - 2\cos^2 x - \cos x - 1 = 0$$

$$-2\cos^2 x - \cos x + 1 = 0$$

$$x = \cos x$$

$$-1[-2x^2 - x + 1 = 0]$$

$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1) = 0$$

$$\cos x = \frac{1}{2}$$

$$x = 60^\circ, 300^\circ$$

$$x = \frac{1}{2}, -1$$

$$\cos x = -1$$

$$x = 180^\circ$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$$

$$20) 4\cos^2 x - 4\sin x - 5 = 0$$

$$4(1 - \sin^2 x) - 4\sin x - 5 = 0$$

$$4 - 4\sin^2 x - 4\sin x - 5 = 0$$

$$-4\sin^2 x - 4\sin x - 1 = 0$$

$$x = \sin x$$

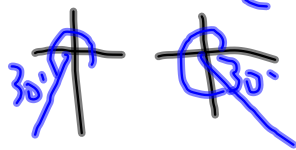
$$-1 \left[-4x^2 - 4x - 1 = 0 \right]$$

$$4x^2 + 4x + 1 = 0$$

$$(2x+1)(2x+1) = 0$$

$$x = -\frac{1}{2}$$

$$\sin x = -\frac{1}{2}$$



$$x = 210^\circ, 330^\circ$$
$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$22) 2 \cos x + \tan x = \sec x$$

$$\cos x \left[2 \cos x + \frac{\sin x}{\cos x} = \frac{1}{\cos x} \right]$$

$$2 \cos^2 x + \sin x = 1$$

$$x \neq 90^\circ, 270^\circ$$

$$2(1 - \sin^2 x) + \sin x - 1 = 0$$

$$2 - 2 \sin^2 x + \sin x - 1 = 0$$

$$-2 \sin^2 x + \sin x + 1 = 0$$

$$x = \sin x$$

$$-1 \left[-2x^2 + x + 1 = 0 \right]$$

$$2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0$$

$$x = -\frac{1}{2}, 1$$

$$\sin x = -\frac{1}{2}$$

$$\cancel{30^\circ} \quad \cancel{330^\circ}$$

$$\sin x = 1$$

$$x = 210^\circ, 330^\circ, \cancel{90^\circ}$$

$$\cancel{90^\circ}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$24) \sin x - \cos x = \sqrt{2}$$

$$\frac{(\sin x)^2 = (\cos x + \sqrt{2})^2}{(\cos x + \sqrt{2})(\cos x + \sqrt{2})}$$

$$\sin^2 x = \cos^2 x + 2\sqrt{2}\cos x + 2$$

$$\frac{1 - \cos^2 x}{-1 + \cos^2 x} = \frac{\cos^2 x + 2\sqrt{2}\cos x + 2}{\cos^2 x - 1}$$

$$2\cos^2 x + 2\sqrt{2}\cos x + 1 = 0$$

$$a = 2$$

$$b = 2\sqrt{2}$$

$$c = 1$$

$$\frac{-2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4(2)(1)}}{2(2)}$$

$$\frac{-2\sqrt{2} \pm \sqrt{8 - 8}}{4}$$

$$\frac{-2\sqrt{2}}{4}$$

$$\cos x = -\frac{\sqrt{2}}{2}$$

$$x = 135^\circ, 225^\circ$$

$$x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$26) \sin \theta - \sqrt{3} \cos \theta = -\sqrt{3}$$

$$\frac{\sin \theta + \sqrt{3} \cos \theta}{(\sqrt{3} \cos \theta + \sqrt{3})(\sqrt{3} \cos \theta + \sqrt{3})} = \frac{\sqrt{3} \cos \theta + \sqrt{3}}{(\sqrt{3} \cos \theta + \sqrt{3})(\sqrt{3} \cos \theta + \sqrt{3})}$$

$$\sin^2 \theta = 3 \cos^2 \theta + 6 \cos \theta + 3$$

$$\frac{1 - \cos^2 \theta}{-1 + \cos^2 \theta} = \frac{3 \cos^2 \theta + 6 \cos \theta + 3}{+\cos^2 \theta - 1}$$

$$4 \cos^2 \theta + 6 \cos \theta + 2 = 0$$

$$x = \cos \theta$$

$$\frac{1}{2} [4x^2 + 6x + 2 = 0]$$

$$2x^2 + 3x + 1 = 0$$

$$(2x+1)(x+1) = 0$$

$$x = -\frac{1}{2}, -1$$

$$\cos \theta = -\frac{1}{2} \quad \cos \theta = -1$$

$$\theta = 120^\circ, 240^\circ, 180^\circ$$

$$28) \sin \theta - \sqrt{3} \cos \theta = 1$$

$$\quad \quad \quad +\sqrt{3} \cos \theta \quad +\sqrt{3} \cos \theta$$

$$(\sin \theta)^2 = (\sqrt{3} \cos \theta + 1)^2$$

$$\quad \quad \quad (\sqrt{3} \cos \theta + 1)(\sqrt{3} \cos \theta + 1)$$

$$\sin^2 \theta = 3 \cos^2 \theta + 2\sqrt{3} \cos \theta + 1$$

$$\begin{array}{r} \downarrow -1 \\ \cos^2 \theta = 3 \cos^2 \theta + 2\sqrt{3} \cos \theta + 1 \\ \uparrow + \cos^2 \theta \quad \quad \quad \uparrow + \cos^2 \theta \quad \quad \quad \downarrow -1 \end{array}$$

$$4 \cos^2 \theta + 2\sqrt{3} \cos \theta = 0$$

$$2 \cos \theta (2 \cos \theta + \sqrt{3}) = 0$$

$$\cos \theta = 0 \quad \quad \quad \cos \theta = -\frac{\sqrt{3}}{2}$$

$$\uparrow \quad \uparrow \quad \quad \quad \uparrow \quad \quad \uparrow$$

$$\theta = 90^\circ, 270^\circ, 150^\circ, 210^\circ$$

$$30) \sin \frac{\theta}{2} + \cos \theta = 1$$

$$\pm \sqrt{\frac{1 - \cos \theta}{2}} + \cos \theta = 1$$

$-\cos \theta$
 $-\cos \theta$

$$\left(\pm \sqrt{\frac{1 - \cos \theta}{2}} \right)^2 = (1 - \cos \theta)^2$$

$(1 - \cos \theta)(1 - \cos \theta)$

$$2 \left[\frac{1 - \cos \theta}{2} = 1 - 2 \cos \theta + \cos^2 \theta \right]$$

$$1 - \cos \theta = 2 - 4 \cos \theta + 2 \cos^2 \theta$$

$-1 + \cos \theta$
 $-1 + \cos \theta$

$$2 \cos^2 \theta - 3 \cos \theta + 1 = 0$$

$$x = \cos \theta$$

$$2x^2 - 3x + 1 = 0$$

$$(2x - 1)(x - 1) = 0$$

$$x = \frac{1}{2}, 1$$

$$\cos \theta = \frac{1}{2}$$

$\theta = 60^\circ$

$$\cos \theta = 1$$

$\theta = 0^\circ$

$$\theta = 60^\circ, 300^\circ, 0^\circ$$

$$32) \cos \frac{\theta}{2} - \cos \theta = 0$$

$$\left(\pm \sqrt{\frac{1+\cos \theta}{2}} \right)^2 = (\cos \theta)^2$$

$$2 \left[\frac{1+\cos \theta}{2} = \cos^2 \theta \right]$$

$$\frac{1+\cos \theta}{-1-\cos \theta} = \frac{2\cos^2 \theta}{-1-\cos \theta}$$

$$2\cos^2 \theta - \cos \theta - 1 = 0$$

$$x = \cos \theta$$

$$2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0$$

$$x = -\frac{1}{2}, 1$$

$$\cos \theta = -\frac{1}{2} \quad \cos \theta = 1$$

$\theta = 120^\circ, 240^\circ$

$$\theta = 120^\circ, 240^\circ, 0^\circ$$

$$34) 13 \cot \theta + 11 \csc \theta = 6 \sin \theta$$

$$\sin \theta \left[\frac{13 \cos \theta}{\sin \theta} + \frac{11}{\sin \theta} = 6 \sin \theta \right]$$

$$13 \cos \theta + 11 = 6 \sin^2 \theta$$

$$\theta \neq 0, 180^\circ$$

$$13 \cos \theta + 11 = 6(1 - \cos^2 \theta)$$

$$13 \cos \theta + 11 = 6 - 6 \cos^2 \theta$$

$$\begin{array}{r} +6 \cos^2 \theta \\ \hline 6 \cos^2 \theta + 13 \cos \theta + 5 = 0 \end{array}$$

$$6 \cos^2 \theta + 13 \cos \theta + 5 = 0$$

$$x = \cos \theta$$

$$6x^2 + 13x + 5 = 0$$

$$(3x+5)(2x+1) = 0$$

$$x = -\frac{5}{3}, -\frac{1}{2}$$

~~$$\cos \theta = -\frac{5}{3}$$~~

$$\cos \theta = -\frac{1}{2}$$

$$\theta = 120^\circ, 240^\circ$$

$$36) 18 \sec^2 \theta - 17 \tan \theta \sec \theta - 12 = 0$$

$$\cos^2 \theta \left[\frac{18}{\cos^2 \theta} - \frac{17 \sin \theta}{\cos^2 \theta} - 12 = 0 \right]$$

$$18 - 17 \sin \theta - 12 \cos^2 \theta = 0 \quad \theta \neq 90^\circ, 270^\circ$$

$$18 - 17 \sin \theta - 12(1 - \sin^2 \theta) = 0$$

$$18 - 17 \sin \theta - 12 + 12 \sin^2 \theta = 0$$

$$12 \sin^2 \theta - 17 \sin \theta + 6 = 0$$

$$x = \sin \theta$$

$$12x^2 - 17x + 6 = 0$$

$$(4x-3)(3x-2) = 0$$

$$x = \frac{3}{4}, \frac{2}{3}$$

$$\sin \theta = \frac{3}{4}$$

$$\left| \begin{array}{c} \sqrt{486} \\ \sqrt{2} \end{array} \right|$$

$$\sin \theta = \frac{2}{3}$$

$$\left| \begin{array}{c} \sqrt{4134} \\ \sqrt{2} \end{array} \right|$$

$$\theta = 48.6^\circ, 131.4^\circ$$

$$41.8^\circ, 138.2^\circ$$

$$38) 16 \cos 2\theta - 18 \sin^2 \theta = 0$$

$$16(1 - 2 \sin^2 \theta) - 18 \sin^2 \theta = 0$$

$$16 - 32 \sin^2 \theta - 18 \sin^2 \theta = 0$$

$$-\frac{1}{2} [-50 \sin^2 \theta + 16 = 0]$$

$$25 \sin^2 \theta - 8 = 0$$

$$\frac{25 \sin^2 \theta + 8}{25} = \frac{8 + 8}{25}$$

$$\sqrt{\sin^2 \theta} = \sqrt{\frac{8}{25}}$$

$$\sin \theta = \pm \frac{\sqrt{8}}{5}$$

~~34.4~~ 34.4 ~~34.4~~

$$\theta = 34.4^\circ, 145.6^\circ, 214.4^\circ, 325.6^\circ$$