

# Additional Trigonometric Identities

## Even and Odd Functions

An *even function* is a function for which  $f(-x) = f(x)$  for all  $x$  in the domain of  $f$ .

An *odd function* is a function for which  $f(-x) = -f(x)$  for all  $x$  in the domain of  $f$ .

### Even Functions

cos

sec

### Odd Functions

sin

csc

tan

cot

Use the even and odd function relationships to find exact values for each of the following.

a.  $\sin(-60^\circ)$

b.  $\cos\left(-\frac{2\pi}{3}\right)$

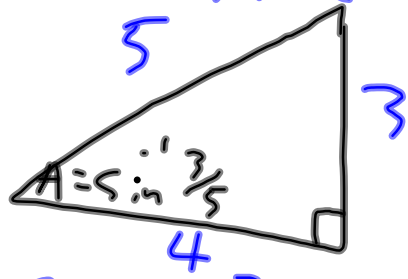
c.  $\csc(-225^\circ)$

# Homework

**Problem Set 5.5 #1-35 odd  
exclude #13 & 23**

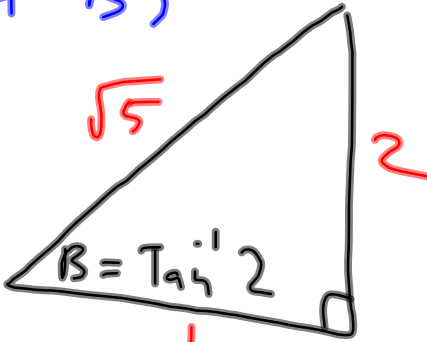
$$1) \sin(\sin^{-1} \frac{3}{5} - \tan^{-1} 2)$$

$\sin(A-B)$



$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$



$$\sin B = \frac{2}{\sqrt{5}}$$

$$\cos B = \frac{1}{\sqrt{5}}$$

$$\sin(A-B)$$

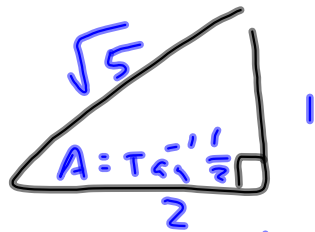
$$\sin A \cos B - \cos A \sin B$$

$$\frac{3}{5} \cdot \frac{1}{\sqrt{5}} - \frac{4}{5} \cdot \frac{2}{\sqrt{5}}$$

$$\frac{3}{5\sqrt{5}} - \frac{8}{5\sqrt{5}} = -\frac{5}{5\sqrt{5}} = -\frac{1}{\sqrt{5}}$$

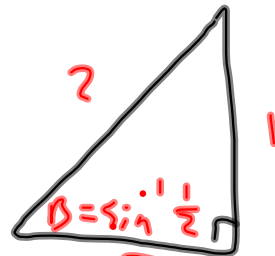
$$3) \cos(\tan^{-1} \frac{1}{2} + \sin^{-1} \frac{1}{2})$$

$$\cos(A+B)$$



$$\sin A = \frac{1}{\sqrt{5}}$$

$$\cos A = \frac{2}{\sqrt{5}}$$



$$\sqrt{3}$$

$$\sin B = \frac{1}{2}$$

$$\cos B = \frac{\sqrt{3}}{2}$$

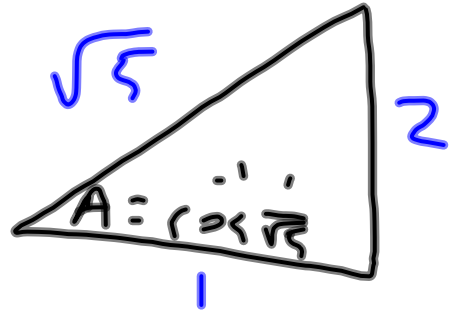
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{5}} \cdot \frac{1}{2}$$

$$= \frac{2\sqrt{3} - 1}{2\sqrt{5}}$$

$$5) \sin\left(2 \cos^{-1} \frac{1}{\sqrt{5}}\right)$$

$\sin 2A$



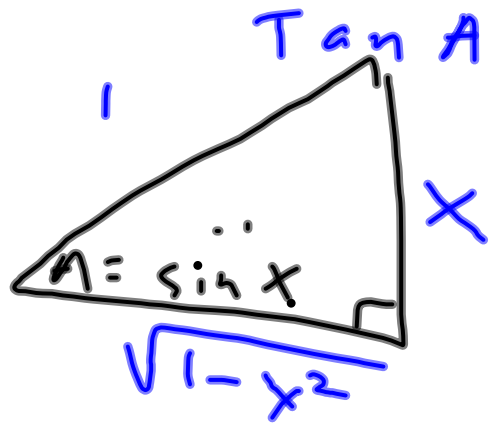
$$\sin A = \frac{2}{\sqrt{5}}$$

$$\cos A = \frac{1}{\sqrt{5}}$$

$$\begin{aligned} \sin 2A &= 2 \sin A \cos A \\ &= 2 \cdot \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} \end{aligned}$$

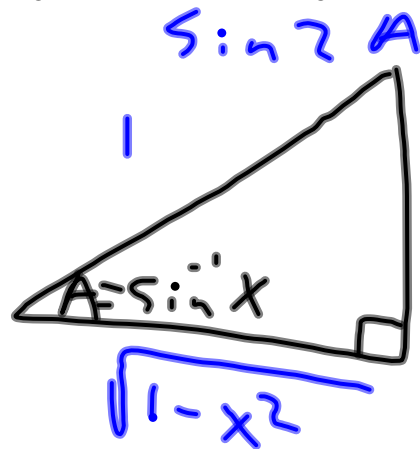
$$\boxed{\sin 2A = \frac{4}{5}}$$

7)  $\tan(\sin^{-1}x)$



$$\tan A = \frac{x}{\sqrt{1-x^2}}$$

$$9) \sin(2 \sin^{-1} x)$$



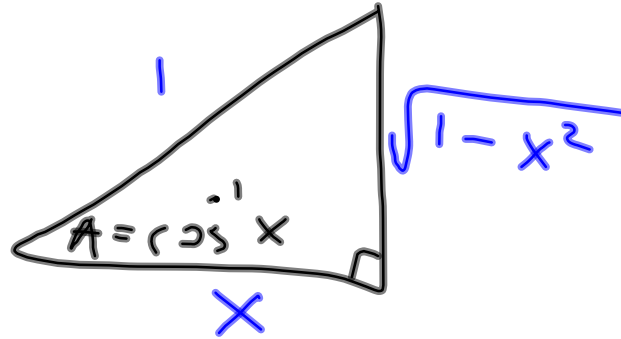
$$\sin A = x$$

$$\cos A = \sqrt{1-x^2}$$

$$\sin 2A = 2 \sin A \cos A$$
$$= 2x\sqrt{1-x^2}$$

$$11) \cos(2 \cos^{-1} x)$$

$\cos ? A$



$$\sin A = \sqrt{1-x^2}$$

$$\cos A = x$$

$$\cos ? A = \cos^2 A - \sin^2 A$$

$$(x)^2 - (\sqrt{1-x^2})^2$$

$$x^2 - (1-x^2)$$

$$x^2 - 1 + x^2$$

$$2x^2 - 1$$

$$15) 10 \sin 5x \cos 3x$$

$$\frac{10}{1} \cdot \frac{1}{2} [\sin(5x+3x) + \sin(5x-3x)]$$

$$5(\sin 8x + \sin 2x)$$

$$5 \sin 8x + 5 \sin 2x$$

$$17) \cos 8x \cos 2x$$

$$\frac{1}{2} [\cos(8x+2x) + \cos(8x-2x)]$$

$$\frac{1}{2} (\cos 10x + \cos 6x)$$

$$\frac{1}{2} \cos 10x + \frac{1}{2} \cos 6x$$

$$19) \sin 60^\circ \cos 30^\circ$$

$$\frac{1}{2} [\sin(60^\circ + 30^\circ) + \sin(60^\circ - 30^\circ)]$$

$$\frac{1}{2} [\sin 90^\circ + \sin 30^\circ] \quad +$$

$$\frac{1}{2} \left[ 1 + \frac{1}{2} \right]$$

$$\frac{1}{2} \left[ \frac{3}{2} \right]$$

$$\boxed{\frac{3}{4}}$$

$$21) \sin 4\pi \sin 2\pi$$

$$\frac{1}{2} [\cos(4\pi - 2\pi) - \cos(4\pi + 2\pi)]$$

$$\frac{1}{2} [\cos 2\pi - \cos 6\pi]$$

$$\frac{1}{2} [\cos 360^\circ - \cos 1080^\circ]$$

$$\frac{2\pi}{1} \cdot \frac{180^\circ}{\pi} = 360^\circ$$

$$\frac{6\pi}{1} \cdot \frac{180^\circ}{\pi} = 1080^\circ$$

$$\frac{1}{2} [1 - 1]$$

$$\frac{1}{2} [0]$$

$$0$$

$$25) \sin 7x + \sin 3x$$

$$2 \sin \frac{7x+3x}{2} \cos \frac{7x-3x}{2}$$

$$2 \sin 5x \cos 2x$$

$$27) \cos 45^\circ + \cos 15^\circ$$

$$2 \cos \frac{45^\circ + 15^\circ}{2} \cos \frac{45^\circ - 15^\circ}{2}$$

$$2 \cos 30^\circ \cos 15^\circ$$

$$\frac{2}{1} \cdot \frac{\sqrt{3}}{2} \cdot \cos(45^\circ - 30^\circ)$$

$$\sqrt{3} [\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ]$$

$$\sqrt{3} \left[ \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \right]$$

$$\sqrt{3} \left[ \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \right]$$

$$\frac{\sqrt{3}}{1} \left[ \frac{\sqrt{6} + \sqrt{2}}{4} \right]$$

$$\frac{\sqrt{18} + \sqrt{6}}{4}$$

$$27) \cos 45^\circ + \cos 15^\circ$$

$$\frac{\sqrt{2}}{2} + \cos \frac{30^\circ}{2}$$

$$\frac{\sqrt{2}}{2} + \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}}$$

$$\frac{\sqrt{2}}{2} + \sqrt{\frac{2 + \sqrt{3}}{4}}$$

$$29) \sin \frac{7\pi}{12} - \sin \frac{\pi}{12}$$

$$\sin 105^\circ - \sin 15^\circ$$

$$2 \cos \frac{105^\circ + 15^\circ}{2} \sin \frac{105^\circ - 15^\circ}{2}$$

$$2 \cos 60^\circ \sin 45^\circ$$

$$\cancel{\frac{2}{1}} \cdot \cancel{\frac{1}{2}} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}}{2}}$$

$$\cancel{\frac{2}{2}} \cdot \frac{\cancel{120}^{15}}{2} = 15^\circ$$

$$31) -\cot x = \frac{\sin 3x + \sin x}{\cos 3x - \cos x}$$

$$-\frac{\cos x}{\sin x} = \frac{2 \sin \frac{3x+x}{2} \cos \frac{3x-x}{2}}{-2 \sin \frac{3x+x}{2} \sin \frac{3x-x}{2}}$$

$$= \frac{\cancel{2 \sin 2x} \cos x}{-\cancel{2 \sin 2x} \sin x}$$

$$-\frac{\cos x}{\sin x} = -\frac{\cos x}{\sin x}$$

$$33) \cot x = \frac{\sin 4x + \sin 6x}{\cos 4x - \cos 6x}$$

$$\frac{\cos x}{\sin x} = \frac{2 \sin \frac{4x+6x}{2} \cos \frac{4x-6x}{2}}{-2 \sin \frac{4x+6x}{2} \sin \frac{4x-6x}{2}}$$

$$= \frac{2 \sin 5x \cos(-x)}{-2 \sin 5x \sin(-x)}$$

$$= \frac{\cancel{2} \cancel{\sin 5x} \cos x}{\cancel{-2} \cancel{\sin 5x} \sin x}$$

$$\frac{\cos x}{\sin x} = \frac{\cos x}{\sin x}$$

$$35) \tan 4x = \frac{\sin 5x + \sin 3x}{\cos 3x + \cos 5x}$$

$$\frac{\sin 4x}{\cos 4x} = \frac{2 \sin \frac{5x+3x}{2} \cos \frac{5x-3x}{2}}{2 \cos \frac{3x+5x}{2} \cos \frac{3x-5x}{2}}$$

$$= \frac{2 \sin 4x \cos x}{2 \cos 4x \cos(-x)}$$

$$= \frac{\cancel{2} \sin 4x \cancel{\cos x}}{\cancel{2} \cos 4x \cancel{\cos x}}$$

$$\frac{\sin 4x}{\cos 4x} = \frac{\sin 4x}{\cos 4x}$$